

## Section 1.1 Problem Solving (Student Assignment #2)

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### Math Talk

A major focus in studying mathematics is to develop and improve one's ability at problem solving. In particular, we want to learn how to solve problems that are quantitative in nature, meaning numbers play an important role in the problem situation. Interesting applications will be used whenever possible to provide a meaningful context for learning and applying mathematical concepts. You will learn how to attack unfamiliar problems by experimenting with many different problem solving strategies such as drawing a picture, constructing a table, and looking for patterns. You will see that problem solving often starts with being able to pick out what information is useful and what can be disregarded. Unlike many traditional "word problems" that give you just the numerical information needed, in the real world you must decide what numbers are relevant to solving the problem. Also, many problems require the solver to research the numerical information that is needed to proceed.

It is essential that you learn that reading, writing, and thinking are integral parts of the problem solving process. You should not expect to quickly read a problem, complete a few quick calculations, and write a single number as your answer. That is not how challenging problems from the real world are solved. To begin to solve a problem you must carefully read and reread the situation until you understand the text, what you're looking for, and what information will help you find it. Then you need to write in your own words the essential information that is given and what you are trying to find. But this is just the beginning; the most challenging part of the process is thinking about a good strategy or plan for solving the problem. Here you must be willing to take the time to think critically about developing your plan. And realize that many problems are not solved in minutes, but take hours, days, months, and even years. Do not be afraid to experiment, to work with other students, and to ask for help. Very few important problems are solved in isolation by only one person.

Now you are ready to carry out your plan, to implement your strategy. Here it is important to be able to explain your work orally and in writing. When you arrive at an answer the final step is to look back and ask yourself if you have answered the question and does your answer make sense in terms of the problem situation. In writing your answer, be sure to use complete sentences and write with the idea that your results need to be understood by your audience. Keep your writing simple, short, and to the point. Act as if you are summarizing your work to someone who knows nothing about the problem. Although you are encouraged to work with others and get extra help when needed, always make sure that you can take ownership of your conclusion. In other words, know of what you write about and take pride in the work being your own.

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It's nice to know you're not alone in learning math and getting acquainted with your classmates is a good beginning. This will help you form study groups that meet regularly outside of class. In the first discovery adventure you will introduce yourself to other students by completing the

handshake problem that follows. This problem is best done in groups of three to four. However you can simulate the problem by using your imagination and drawing a picture.

**Discovery Adventure 1: Getting Acquainted**

If possible form a small group of 3-4 students. However you can still answer the questions below if you are working alone. To get acquainted there will be one and only one introduction (handshake and exchange of names) between you and each person in your group. Do not shake hands with any student twice! Now perform the introductions within your group, and then answer the questions below.

- a.* Imagine you arrive to class early and the room is empty of people. How many introductions are possible?
- b.* In walks another student. How many introductions are possible among two people? Remember, name exchange and handshake will occur only once between any two individuals.
- c.* A third student enters the room. How many introductions are possible among three people?
- d.* A fourth student arrives to class. How many introductions are possible among four people?
- e.* A fifth student arrives to class. How many introductions are possible among five people?
- f.* Is there are pattern between the number of students and the number of introductions? Complete the table below. Look for a pattern among the numbers that will allow you to immediately calculate the number of introductions occurring among 6, 7, or 20 people.

Students	1	2	3	4	5	6	7	...	20
Introductions								...	

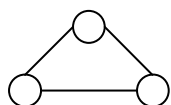
- g.* Suppose  $n$  represents the number of students present. These  $n$  students are standing in a circle when one of them steps forward and then shakes the hands of all the students remaining in the circle. How many hands does this one student shake?
- h.* Now suppose each of the  $n$  students in the circle takes a turn stepping forward and then shakes the hands of all the students remaining in the circle. How many handshakes have taken place after all students have taken their turn? *Hint:* Repeat handshakes have occurred.
- i.* Can you think of a formula using  $n$ , which will calculate the number of introductions among  $n$  people without any repeat handshakes occurring? In other words, represent the number of introductions among  $n$  people where the name exchange and handshake occur only once between any two individuals.

## Feedback

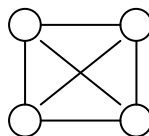
A nice way to answer questions *a-e* is to form a group of 4 and actually go through the introduction process with 2, 3, and 4 people. The group can then think about how many additional handshakes will occur each time another person is added to the group. As progressively more people are added, the new person shakes hands with everyone in the group and this amount is added to the previous total. This pattern can be seen in the table below.

Students	1	2	3	4	5	6
Introductions	0	1	3	6	10	15
Sum	$1 + 0$	$2 + 1$	$3 + 3$	$4 + 6$	$5 + 10$	

Constructing a table to organize your information is a common problem solving strategy to use when looking for patterns in numbers. Another well used problem solving strategy is drawing a picture of the situation. For example, if you were working alone on Discovery Adventure 1, circles and lines could be used to represent people and handshakes. The figure below demonstrates how you can count the number of introductions among a group of 3 or 4 people. However, as more circles (people) are added, the additional lines (handshakes) become increasingly difficult to clearly see and count.



3 students  
3 handshakes



4 students  
6 handshakes

If the letter  $n$  represents the number of students present and one of the  $n$  students steps forward and then shakes the hand of all the students remaining, then  $n - 1$  handshakes will occur. If each of the  $n$  students takes a turn at stepping forward and then shakes the hand of all the students remaining, then  $n \cdot (n - 1)$  handshakes will occur after all students have taken their turn.

However, repeat handshakes will have occurred among each pair of students. Therefore, we divide the result by 2 to obtain the formula that calculates the number of introductions among  $n$  people without any repeat handshakes occurring. That formula is given below.

$$\left. \begin{array}{l} \text{Number of unique introductions} \\ \text{among } n \text{ people} \end{array} \right\} = \frac{n \cdot (n - 1)}{2}$$

The letter  $n$ , which represents the number of students, is called a **variable**. Here the value of  $n$  can be any number from the set of counting numbers 1, 2, 3... depending on how many people are present. We will explore additional meanings and uses of variables in upcoming sections of this text.

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In the next Discovery Adventure, you will begin to develop an organized way to approach problem solving.

### **Discovery Adventure 2: Money for College**

It's June 2 and Nina Carey at age 23 has just decided to start College. She will take 13 credit hours at the local community college during the upcoming fall semester. The college catalog indicates that tuition and fees are \$120 per credit hour to register for classes and she will need a textbook for each of her 4 classes. Currently Nina has no money saved, but will start working next week for a company that pays her \$9.00 per hour, with no deductions, and allows her to work anywhere from 15 to 30 hours per week. She lives at home with her parents who expect her to contribute \$100 per week to help out with household expenses. School starts in 12 weeks and Nina worries about not be able to afford college because she alone is responsible for the cost.

- a.* What must Nina figure out or *find* to ease her financial worries?
- b.* Can you think of any other quantitative information not listed above that might help Nina?
- c.* Make a bullet list of any *given* information that will help Nina plan the best course of action.
- d.* Demonstrate any calculations needed to assure Nina that she will have enough money for college.

- e. Write a summary or conclusion that explains whether Nina can afford college in 12 weeks.

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## Feedback

The main question to be answered in this problem is whether Nina can afford the cost of College. A sub question is how many hours she must work to pay her expenses. While the questions might be clear in this example, in other situations it might not be so obvious. If you cannot express your understanding of the problem in words, then you probably need to spend more time reading and thinking about the problem situation. Once you understand what you are looking for, the next step is to organize all the relevant information.

Let's examine a general problem solving procedure that can guide you in your quest to answer the questions in any problem situation. Discovery Adventure 2 will be used as an example of applying this method. Here are 5 steps that can help you begin to develop a problem solving method of your own.

## The 5-step Problem-Solving Process

### I. Read and Understand

You must read and reread the problem situation until it is clear what the problem requires. Familiarize yourself with the problem until you get the big picture. Do not concern yourself with any details until you understand its purpose. Then organize the essential information by writing what you are trying to find and any given information that will help in the problem-solving process. For example, in Discovery Adventure 2 you might write the following.

**FIND:** Number of hours Nina must work during the summer to afford college.

**GIVEN:**

- Register for 13 credit hours
- Tuition and fees are \$120 per credit hour
- 4 textbooks needed
- Job pays \$9.00 per hour
- Can work 15-30 hours per week
- Owes parents \$100 per week
- 12 weeks to save money

In this example you are not given the cost of textbooks or whether Nina might need any extra spending money throughout the summer for gas, food, cloths, etc. In the real world you will often have to do some research activity to gather all the necessary information that allows you to proceed with the problem solving process.

In the read and understand step, the most important parts of a problem are:

- Identifying the unknown (what you are try to find)
- Listing the useful data (numerical information given or found through research)
- Stating the conditions that must be satisfied

## **II. Plan a strategy**

You need to develop a game plan on how to approach the problem. This is the hardest part of the process. Consider what strategies you might implement, be willing to experiment with any idea, and approach the problem from different angles. If you get stuck, take a break or ask for help. Some strategies you can try are given next.

Visual Strategies:

- Draw a Picture
- Make a Graph

Numerical Strategies:

- Guess and Test
- Create a Table
- Look for Patterns
- Perform Arithmetic Operations

Algebraic Strategies:

- Use a Variable to Represent the Unknown
- Create an Equation
- Find a Formula

Other Strategies:

- Find a similar problem that has been solved
- Look up a definition
- Break the problem into smaller parts
- Work backwards
- Unit analysis

This list is far from complete, but it presents you with some useful strategies to try. The best way to improve at problem solving is to work as many problems as possible. The more you practice the easier it becomes. In Discovery Adventure 2 a good strategy is to perform the arithmetic operations that will give Nina's income and expenses for the next 12 weeks. All this information can then be organized in a table for easy viewing.

## **III. Implement a strategy**

In this step you put into action any strategy that is part of your plan, such as completing a table, drawing a picture with labels, or performing a calculation. Often you will need multiple strategies in finding your solution. Make sure you are clearly showing all your work in an organized manner. The calculations and table for Discovery Adventure 2 follows.

Expenses:

- Tuition =  $(13 \text{ cr}) \cdot (120 \text{ \$/cr}) = \$1560$
- Books =  $4 \cdot (\$100) = \$400$  (this is an estimate)
- Parents Payment =  $(12 \text{ wk}) \cdot (100 \text{ \$/wk}) = \$1200$

Income:

- Weekly Income =  $(9 \text{ \$/hr}) \cdot (30 \text{ hr}) = \$270$
- Total Income =  $12 \cdot (\$270) = \$3240$

Expenses		Income	
Tuition	\$1560	Weekly	\$270
Books	\$400		
Parents Payment	\$1200		
Total	\$3160	Total (12 weeks)	\$3240

#### IV. Check your results

First ask yourself if your results seem reasonable and make sense in terms of the problem situation. For example, in Discovery Adventure 2, \$1560 for tuition and fees seems like a reasonable amount if you take 13 credits at \$120 per credit. Also, check your answer(s) by making sure all mathematical work was done correctly. Sometimes this involves the use of a calculator, but mental estimation is just as important. For example, since 13 is the sum of 10 and 3 we can multiply \$120 by 10 which just adds a zero giving \$1200. Then 3 times \$120 is \$360 and \$1200 plus \$360 is \$1560.

#### V. Conclude and Summarize in writing

The results you obtain should be expressed in words using complete sentences. You need to make it clear to your audience what you have found and what conclusions you have reached. For example, in answering part *e* of Discovery Adventure 2, instead of just stating whether Nina can afford to pay for college, you might summarize as follows.

If Nina works 30 hours per week for 12 weeks, then she will earn \$3240. Assuming her books cost a total of \$400, her expenses will be \$3160. Therefore, since her income is \$80 more than her expenses, Nina should have enough to pay for her fall semester at college. This conclusion assumes there are no additional expenses occurring.

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A person's financial situation often determines their ability to afford college. However, a person's time management skill is a major factor in their success as a student. In the next discovery adventure you will use the 5-step problem solving process to help Nina manage her time between school, work, and life's many demands.



**Discovery Adventure 3: Time to Study**

It's the start of the fall semester and Nina, has just managed to pay her school bills. Now her worries turn to having enough time to complete her school work. She wants to be a successful student but all her instructors expect their students to complete 2 hours of homework for every 1 hour spent in class. Nina will be in class 13 hours per week, she will exercise 1 hour each day, spend 8 to 10 hours per week in her car commuting, and hopes to spend at least 4 hours per day eating and relaxing. She will continue to work at the same job and has the option of working anywhere from 15 to 30 hours per week. Also, Nina requires 8 hours of sleep each night to function at a high level. How many hours per week should Nina work at her job?

- a.** If you have read the problem enough times to understand the situation, then state what you must find and the given information that will be useful in finding it.

Find:

Given:

- b.** Plan a strategy to solve this problem and implement that strategy below.

- c.** Check your results, and then summarize your answer by giving advice to Nina.

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## Feedback

This problem is an open ended question, meaning there is more than one possible answer depending on what assumptions are made. For example, the rule of thumb for homework, complete 2 hours of homework for every 1 hour spent in class, is just an estimate. The time spent on homework can depend on the subject and the student's ability. Also, Nina's commuting time in her car is an estimate. Should she go with the longer estimate of 10 hours or average the time to 9 hours, or perhaps we can leave earlier or take a different route. Does Nina really need 8 hours of sleep every night? Can she spend less time eating and relaxing? And what is the minimum amount of hours she can work and still pay all her expenses? Can we assume that the fall will be similar to the summer in terms of income and expenses?

Although not all the information we are given is concrete, the approximations should be good enough to arrive at some reasonable conclusion. However, it would be helpful to research the amount of income Nina needs to cover all her expenses. Let's start to solve this problem by writing what we are trying to find and listing all the given information that is relevant.

**Find:** Number of hours per week to work at job

**Given:**

- 2 hours of homework for 1 hour in class
- In class 13 hours per week
- Exercise 1 hour per day
- Commute 8-10 hours per week
- Eat and relax 4 hours per day
- Sleep 8 hours per day
- Work at job 15-30 hours per week

Our strategy first involves calculating the number of hours in a week and the number of hours Nina is occupied in activities other than her job. The difference between these two values will give us the number of hours she could possibly work at her job.

### Implement a strategy

$$\text{Time in a week} = (7 \text{ days}) \cdot (24 \text{ hr/day}) = 168 \text{ hours}$$

Nonworking time per week: In class = 13 hours

$$\text{Homework} = (2 \text{ hr/class hr}) \cdot (13 \text{ class hr}) = 26 \text{ hours}$$

$$\text{Exercise} = (1 \text{ hr/day}) \cdot (7 \text{ days}) = 7 \text{ hours}$$

$$\text{Commute} = 10 \text{ hours (Assuming higher time)}$$

$$\text{Sleep} = (8 \text{ hr/day}) \cdot (7 \text{ days}) = 56 \text{ hours}$$

$$\text{Total nonworking time} = 13 + 26 + 7 + 10 + 56 = 140 \text{ hours}$$

$$\text{Difference} = \text{Hours in week} - \text{total nonworking hours}$$

$$= 168 - 140$$

$$= 28 \text{ hours available}$$

We finish by summarizing our results for Nina in the following conclusion

**Conclusion**

According to the given information Nina has 28 hours available to work at her job.

However, this leaves no extra time for any other obligations that may arise. While there is no definite answer, perhaps 20-25 hours might be reasonable depending on how much income she needs to cover her expenses.

As was stated previously, developing good problem solving strategies comes with practice. The exercises that follow provide many different opportunities to improve. Refer to the 5-step problem-solving process (page 5) as a guide on how to proceed.

**Exercises for Section 1.1**

1. Return to Discovery Adventure 2 and reread the situation. Now suppose Nina works 30 hours per week during the 12 week summer period. She calls the book store and finds the following information about the cost of her books.

- History \$115
- Math \$84
- English \$78
- Computer \$107

Nina also calculates that gas will cost her \$20 per week and she decides to allow herself \$25 per week as extra spending money.

- a. Show that Nina will not be able to afford college using the new information given above.
- b. If her boss decides to let her work more than 30 hours, then how many hours must she work per week to have enough money to pay for school?
2. Return to Discovery Adventure 3 and reread the situation. The first week of classes is over and Nina realizes that her checking account is almost empty. Now suppose Nina has 18 weeks before she must have tuition, fees, and book money for the spring semester. She still gives her parents \$100 per week and has found she will need \$20 per week for gas and \$25 per week as extra spending money. Nina expects her school bills to be the same as the fall semester. How many hours does she need to work to just cover all her expenses?

3. The Stratham Rocks baseball team is thrilled to have won 24 out of their first 40 games played. How many of the 48 remaining games must the Rocks win to keep the same wins to losses ratio? There are many ways to solve this problem, so be sure to show all your work in an organized manner and explain the method used to arrive at your answer.
  
4. Professor Quirk teaches two different sections of Introductory Algebra on Monday, Wednesday, and Friday. He has just finished grading the quiz given in his 8:00 a.m. class (section 01). In this section, 20 out of 32 students passed the quiz. How many of the 40 students in his 11:00 a.m. class (section 03) must pass to keep the same pass to fail ratio?
  
5. Suppose you have agreed to take three children ages 10, 12, and 15 to a baseball game. The two youngest are girls that are not big fans of baseball but they are still excited about attending the game. The oldest is a boy knows all the players on the team and follows the team's fortunes on a regular basis. There are three categories of seats to choose from. Box seats cost \$75 per ticket, grandstand seats cost \$40 per ticket, and bleacher seats cost \$22 per ticket. A friend who attended a game two years ago says you can expect to pay \$4.50 for a hotdog, \$3.50 per drink, and \$2.50 for chips. Also, he claims that the parking cost him \$2.00 per hour. Although you want the kids to have a good time you can only afford to spend \$200 on this entire outing. So you wonder, what are the best seats to purchase?
  - a. Can you think of any other costs involved in making this trip to the ballgame?

- b.** Write what you're trying to find and list all the given information that seems important.

FIND:

GIVEN:

- c.** Explain your plan or strategy for solving this problem.

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- d.** Implement your plan or strategy below in a clear, organized manner.

- e.** Write your conclusion using complete sentences.

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- 6.** Jim is planning a 6-day, 5-night family vacation at Disney World with his wife Sue and their three children Michael, Megan, and Marissa. Jim and his family can stay in a 4-star hotel for \$180 per night, a 3-star hotel for \$110 per night or a 2-star hotel for \$60 per night. They can get a direct roundtrip flight to Orlando for \$220 per person or a flight with one stop for \$183 per person. After talking with a friend who went to Disney 2 months ago he knows that \$1200 should cover the food for the trip. If Jim has \$3000 saved for the trip and wants his family to have the best vacation possible, then what options should he choose for air travel and hotel?

**a.** Can you think of any other costs involved in making this trip?

**b.** Write what you're trying to find and list all the given information that seems important.

FIND:

GIVEN:

**c.** Explain your plan or strategy for solving this problem.

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**d.** Implement your plan or strategy below in a clear, organized manner.

*e.* Write your conclusion using complete sentences.

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**7.**