Vectors - Physics vs. Math

Physics

- Emphasize physical applications
- Set up problem to simplify analysis

Mathematics

- Emphasize geometrical applications
- Analyze the most general case

When we do the same problem,
we often use different notation!

Vectors Notation

Physics

- Notations

  $\vec{A}$ $\mathbf{A}$ $\mathbf{v}$ is velocity

- Unit vectors
  - shows direction; length 1

  $\hat{i}$ $\hat{j}$ $\hat{k}$ $\hat{x}$ $\hat{y}$ $\hat{z}$ $\hat{r}$ $\hat{\theta}$ $\hat{\varphi}$

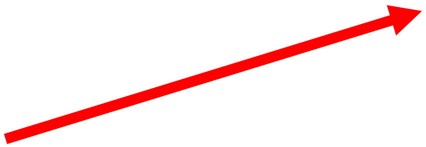
Mathematics

- Notation

  $\overrightarrow{AB}$ $\mathbf{v}$ is vector

- Unit Vectors
  - normalizes a vector **u** = **v**/||**v**||

  $\vec{i}$ $\vec{j}$ $\vec{k}$



Vector Description

Physics

- Vector is an arrow
  - Direction
  - Magnitude (Amount)
- Vector terms
  - Head
  - Tail
- Graphical

Mathematics

- Vector is an list
  - Direction
  - Length
- Vector terms
  - Initial point
  - Terminal point
- Analytical
  - $\mathbf{v}=P_1P_2= \langle x_2-x_1,y_2-y_1$

Vector Units

Physics

- All vectors have units
  - Examples
    - N
    - m/s
  - Typical vector:

$$\vec{d} = 4.0\text{m}\ \hat{i} + 7.2\text{m}\ \hat{j}$$

Mathematics

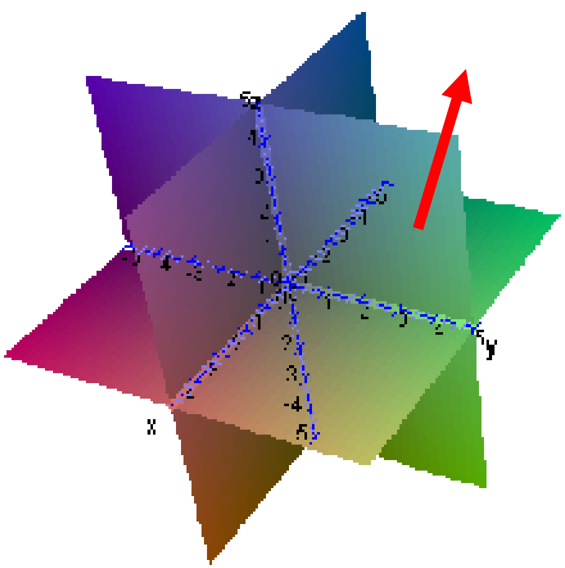
- Pure vectors don't have units
  - In applied examples
    - N
    - m/sec
  - Typical vector:

$$\vec{v} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k}$$

Vector - Physical Quantities

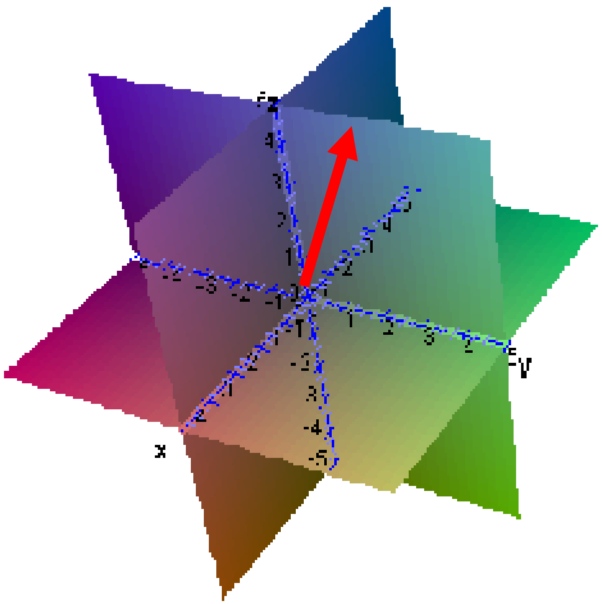
Physics

- Tail of vector is at point of application



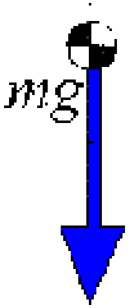
Mathematics

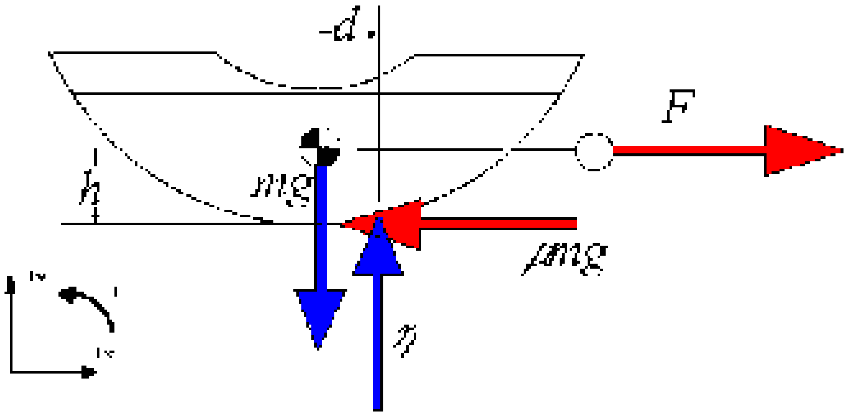
- Tend to put tail at the origin



Vector Physical Quantities

In Physics the force of gravity vector is shown at the center of gravity.





Eddie Bart, Rhodes University, South Africa

Vector Algebra

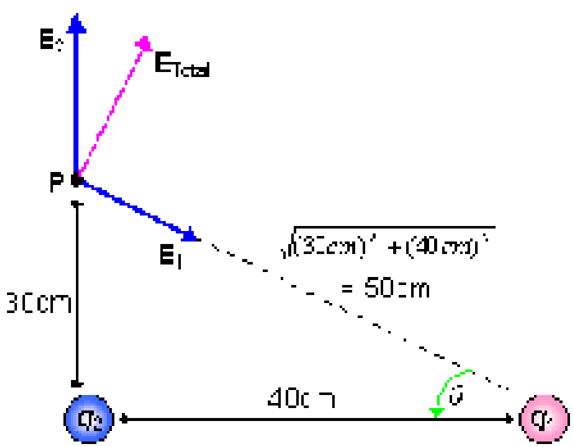
Physics

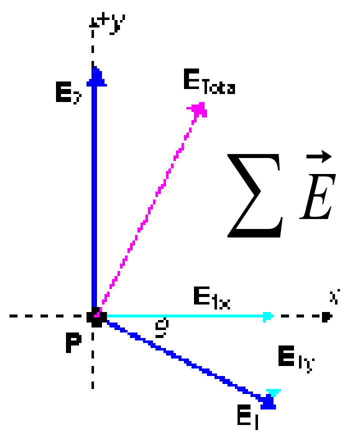
- Vectors usually given in terms of magnitude and direction
  - Students need to find components first

Mathematics

- Vectors often given in terms of components

$$\vec{v}_1 + \vec{v}_2 = \left(\vec{v}_{1x} + \vec{v}_{2x}\right)\,\vec{i} + \left(\vec{v}_{1y} + \vec{v}_{2y}\right)\,\vec{j}$$





$$\sum \vec{E} = \left(E_{1x} + E_{2x}\right)\,\hat{i} + \left(E_{1y} + E_{2y}\right)\,\hat{j}\;\; N/C$$

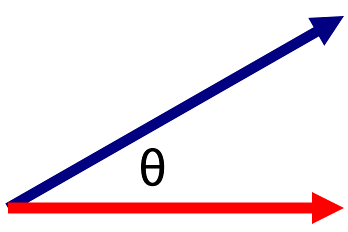
Dr. Joseph Howard, Salisbury U., Australia

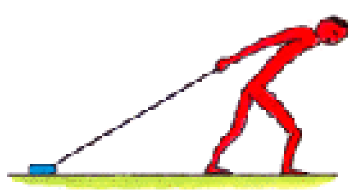
Vectors - Dot Product

Physics

- The dot product of vectors
  - the angle is known
  - answer is a scalar quantity

$$W = \vec{F} \cdot \vec{s}$$





Mathematics

- The dot product of vectors
  - used to find the angle between vectors

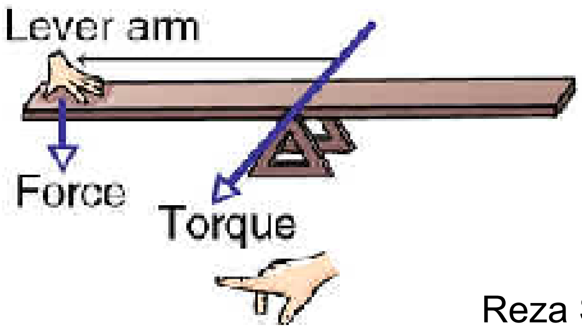
$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|}$$

Vectors - Cross Product

Physics

- The cross product of vectors
  - the angle is known
  - answer is a vector quantity

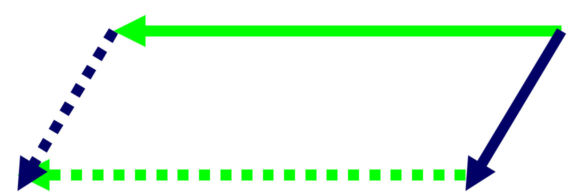
$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta$$



Mathematics

- The cross product of vectors
  - used to find the area of a parallelogram with vectors as the two sides

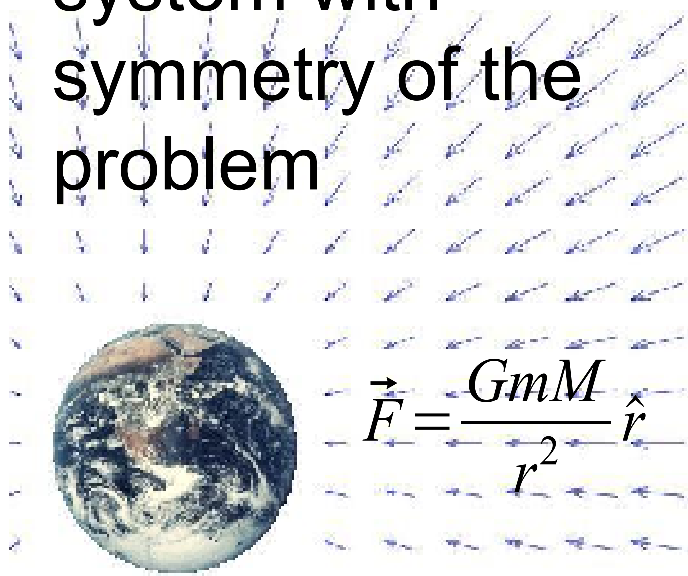
$$\|\vec{u} \times \vec{v}\| = \|u\|\|v\| \sin\theta$$



Vector Fields

Physics

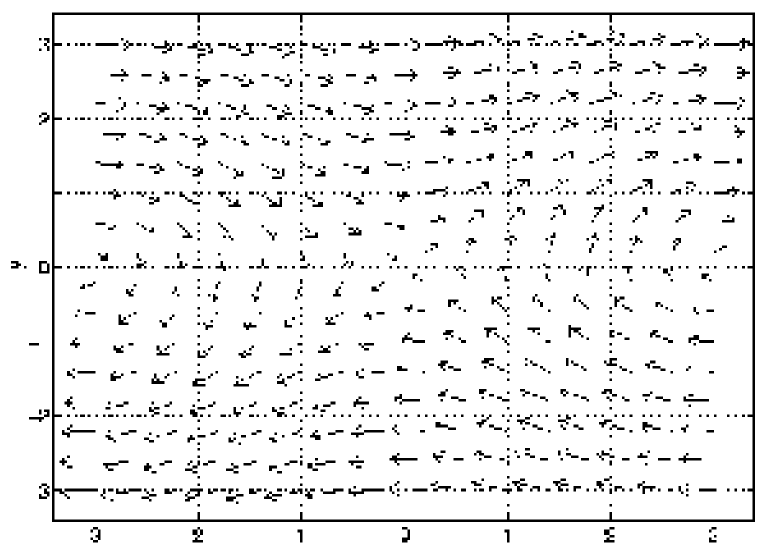
- Symmetrical cases that recur
- Match coordinate system with symmetry of the problem



$$\vec{F} = \frac{GmM}{r^2}\hat{r}$$

Mathematics

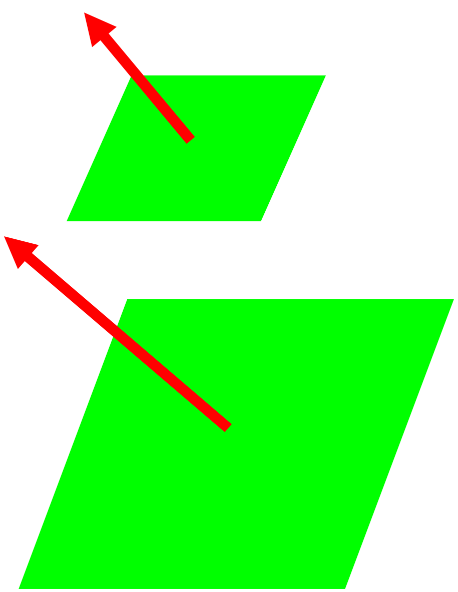
- Look at exceptional cases also

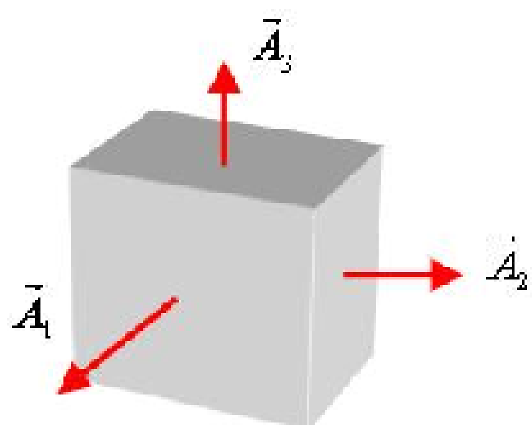


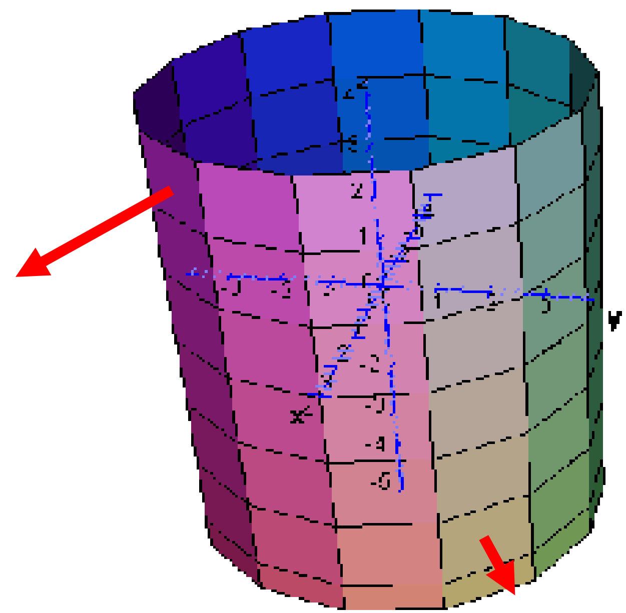
Area vector examples

Vectors

- An area may be represented by a vector
  - Vector direction is perpendicular to the area
    - Vector points out on a closed surface
  - Vector magnitude corresponds to area size







Remediation

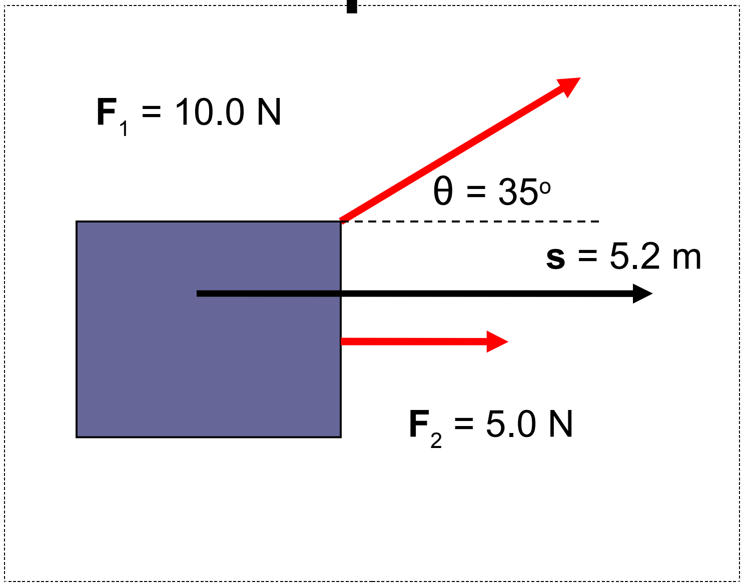
Possible remediation steps:

- Students assigned translation of problems
- Use both notations
- Particular topic threaded throughout calculus course

Specific Topics

Topic	Example
vectors	force vector, position vector
cross product	torque (moment)
derivatives/ gradient	velocity/ energy conservation
vector fields	force fields
line integral	work with a variable force

Sample Translation Problem



A crate experiences two forces applied as shown in the diagram. As a result the crate slides a distance of 5.2 m. How much work is done on the crate?

Physics Approach

In general, with more than one force acting, one way to calculate the total work is to consider the work done by each force separately:

$$W = \left(\sum \vec{F}\right)\cdot\vec{s}$$

$$W = \sum W = F_1 s\cos\theta_1 + F_2 s\cos\theta_2$$

$$= (10.0\ N)(5.2\ m)\cos(35^o) + (5.0\ N)(5.2\ m)\cos(0^o)$$

$$= 45.1\ Nm + 25\ Nm$$

$$= 70\ J$$

Math Approach

In general, with more than one force acting, the force vectors can be added together, and then the dot product computed.

$$W = \vec{F}\cdot\vec{s}$$

$$W = \vec{F}\cdot\vec{s} = Fs\cos\theta$$

To sum the forces, the components are first determined:

$$\vec{F}_1 = 10\cos(35^o)\,\vec{i} + 10\sin(35^o)\,\vec{j} \qquad \vec{F}_2 = 5\ \vec{i}$$

$$\vec{F}_1 + \vec{F}_2 = 8.19\ \{\vec{i} + 5\vec{i} + 5.74\ \{\vec{j}\,\text{¿} = 13.19\ \{\vec{i}\,\text{¿} + 5.74\ \{\vec{j}\,\text{¿¿}$$

$$W = \left(\sum \vec{F}\right)\cdot\vec{s} = \langle 13.19,\ 5.74\rangle\cdot\langle 5.2,\ 0\rangle \qquad W = 70\ Nm$$